

Arbitrarily Sampled Fourier Transform for 5D Interpolation

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Summary

We propose the **Arbitrarily Sampled Fourier Transform (ASFT)** method for 5D interpolation. ASFT is based on the Fourier theory in the $f-k^4$ domain. Comparing to other algorithms in the class, ASFT is able to achieve better sparsity in the $f-k^4$ domain. Real data examples from Western Canadian Sedimentary Basin (WCSB) show that ASFT produces excellent interpolation results.

Introduction

Seismic trace interpolation, which spatially transforms irregularly sampled acquired data to regularly sampled data or to any desired grid in general, is an important step in seismic data processing. A class of algorithms, such as Minimum Weighted Norm Interpolation (MWNI), Projection Onto a Convex Set (POCS), Anti-Leakage Fourier Transform (ALFT), and Matching Pursuit (MP), are based on the Fourier theory in the $f-k^4$ domain by computing the estimated spatial frequency content of irregularly sampled data.

Theory

All the Fourier based interpolation schemes try to first estimate the spatial frequency content distributions in the $f-k^4$ domain. This is done either by snapping traces to the closest “bin” position and then applying FFT, such as in POCS, or by a more elaborated method such as computing weighted DFT in ALFT.

Then either a cut-off threshold is applied such that only the spatial frequencies with energy greater than the threshold are kept, or only the largest spatial frequency contents are selected. The former approach could cause leakage so newer methods such as ALFT and MP use the latter approach.

However, ALFT and MP only estimate the frequency contents at regular grid points in the $f-k^4$ domain, so their selection could be suboptimal as the spatial frequency with the highest energy could lie at an arbitrary point in the $f-k^4$ domain.

ASFT addresses the two problems above by iteratively solving a gradient-based optimization problem for accurate frequency representation. As a result, for ASFT, true positions of the input traces are used for computation, and the spatial frequency content is allowed to be at an arbitrary point in the $f-k^4$ domain, as summarized in the table.

	Uses exact input trace positions	Allows arbitrary spatial frequency in the $f-k^4$ domain
MWNI	○	○
POCS	○	○
ALFT	●	○
MP	●	○
ASFT	●	●

Examples

A typical work flow is shown in Figure 1. The input data are CMP gathers with deconvolution, statics, scaling and final velocity applied. Noise attenuation should be applied to the input data as well for optimal interpolation output. The gathers are sorted into Common Offset-Azimuth (COA) domain or Common Offset Vector (COV) domain as processors' preference. At the first, 1-D FFT is applied to each trace to transform the data from time domain to frequency domain. Then, data at each temporal frequency is transformed by ASFT into 4-dimensional wavenumber domain using the original spatial coordinates, not the bin centre coordinates (important). Interpolation is applied to each temporal frequency slice in 4-dimensional wavenumber domain. After solving all uneven spaced wavenumbers in K4 domain, they are inverse transformed back to temporal frequency domain by ASFT and further transformed back to time domain by FFT. Figure 2 shows a CMP gather before and after interpolation. Interpolation output has more traces than the input gather as requested. The characters of the input gather are naturally preserved after interpolation.

We have applied ASFT interpolation to a dataset in Western Canadian Sedimentary Basin (WCSB). It is a mega bin dataset with 2:1 aspect ratio, in other words, every second inline is empty if the bin is a square. The task for interpolation with this dataset is: a. regularizing data in azimuth and offset direction; b. filling the empty inlines. Figures 3 – 5 show the inline, xline and time slice comparisons before and after interpolation. The inlines before and after interpolation are almost identical, because the inline showing here is a live inline in the input data. For this inline, all interpolation did is regularizing azimuth and offset. The slight difference between the before and after sections is caused by different offset/azimuth distribution, since the distribution is not uniform before interpolation.

As mentioned above, every second trace in an xline is empty. For easy comparison, we poststack interpolated the stack of the uninpolated data (picture on the left of Figure 4). The section on the right side (after ASFT interpolation) has more details compared to the section with poststack interpolation, or less smearing.

Time slice and vertical sections show that the geological features are well preserved after interpolation. Xline section shows that ASFT handles mega bin geometry (upsampling) properly.

Conclusions

The proposed ASFT method is able to use exact input traces positions and allows any spacial frequency content, and is able to achieve better sparsity in the f-k4 domain.

From the examples it can be seen that ASFT effectively interpolates seismic traces and preserves geological structures.

Acknowledgements

We would like to thank an anonymous company for letting us use the data for illustrations. We also thank John Chiu, Don Gee and Laurie Ross at Geo-X for processing the data.

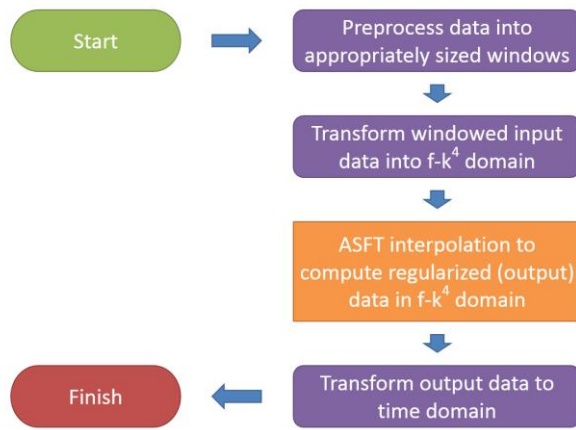


Figure 1. Processing flowchart of the ASFT Method.

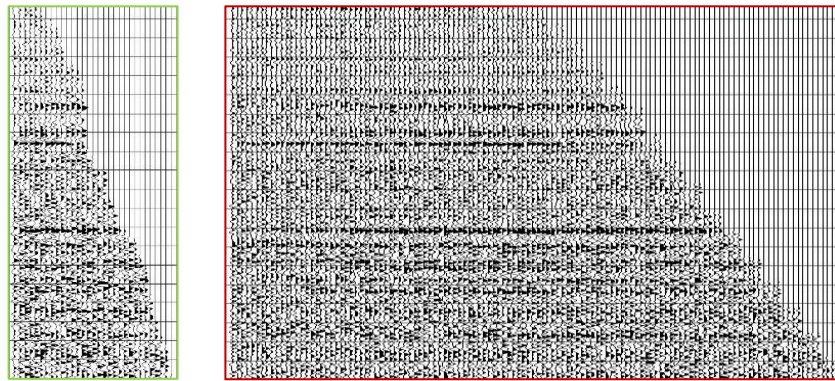


Figure 2. Comparison of gather traces before and after interpolation.

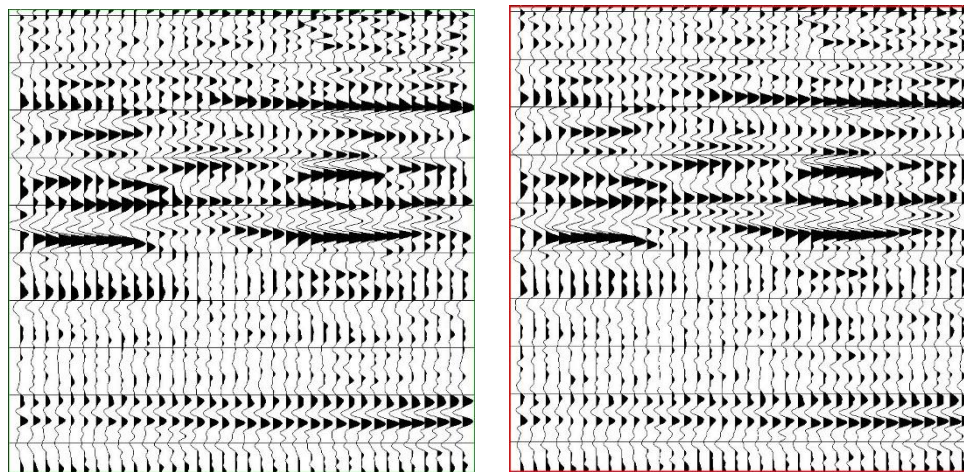


Figure 3. Comparison of inline stack before (left) and after (right) interpolation.

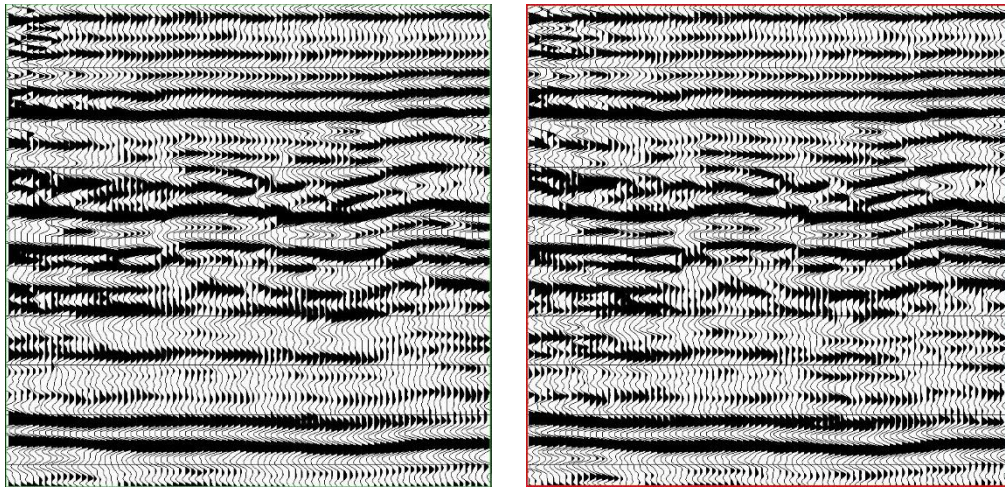


Figure 4. Comparison of xline stack before (left) and after (right) interpolation. Since every second trace in an xline is dead before interpolation, the picture showing on the left and poststack interpolated, just for easy comparison.

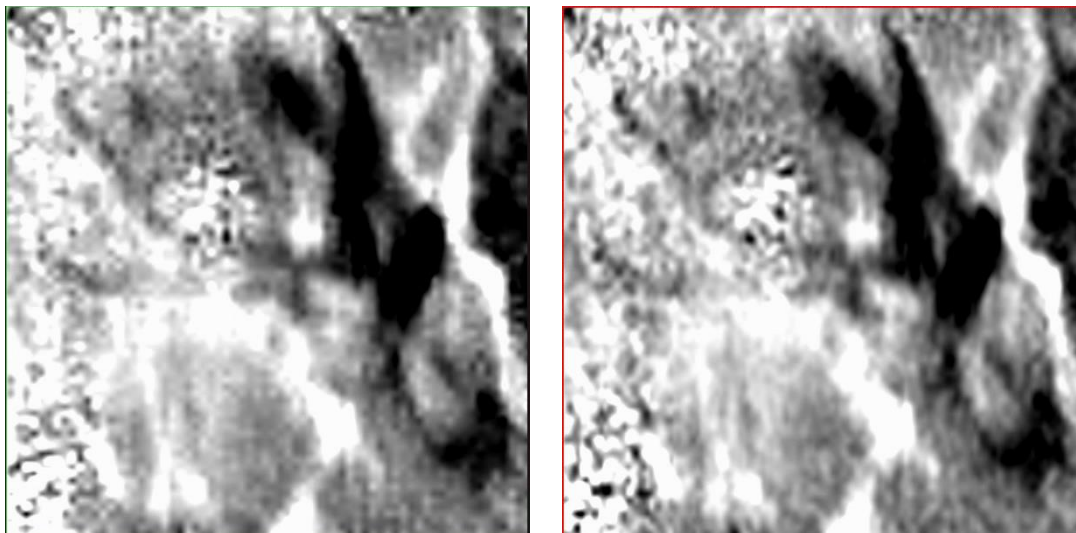


Figure 5. Comparison of time slice before (left) and after (right) interpolation.

References

- Abma, R., and N. Kabir, 2006, 3D interpolation of irregular data with a POCS algorithm: *Geophysics*, 71, no. 6, E91–E97, doi:10.1190/1.2356088.
- Liu, B., and M. Sacchi, 2004, Minimum weighted norm interpolation of seismic records: *Geophysics*, 69, 1560–1568, doi:10.1190/1.1836829.
- Tropp, J. A., Anna, and C. Gilbert, 2007, Signal recovery from random measurements via orthogonal matching pursuit: *IEEE Transactions on Information Theory*, 53, 4655–4666, doi:10.1109/TIT.2007.909108.
- Xu, S., Y. Zhang, D. Pham, and G. Lambaré, 2005b, Antileakage Fourier transform for seismic data regularization: *Geophysics*, 70, no. 4, V87–V95, doi:10.1190/1.1993713.