Seismic data Interpolation in the Continuous Wavenumber Domain, Flexibility and Accuracy

Ye Zheng
Geo-X Exploration Services Inc.

Summary

Our recently developed algorithm, ASFT (Arbitrarily Sampled Fourier Transform), allows interpolating seismic data without snapping the input traces into bin centers and solves the Fourier components in the continuous wavenumber domain, which means the solved Fourier components are not restricted to a predefined grid as in other algorithms. This algorithm preserves amplitude and phase of the input data and does not smear AVO and anisotropic responses of the data. Another feature of this algorithm is that the output locations after interpolation are flexible, e.g. it can have more azimuthal sectors at far offset than at near offset so that each output trace covers approximately the same area in the azimuth-offset domain. A real data example will be presented.

Introduction

In theory, the requirement of seismic acquisition is that the seismic wave field should be sampled in even intervals both in time and space with the sampling frequency at least two times higher than signal frequency. However, in practice, this will never be satisfied in the space domain, because i) it costs too much; ii) field accessibility and environmental restriction prevent deploying receivers and sources in some areas. Multi-dimensional interpolation is a tool to make the less ideally sampled data satisfy some processing algorithm requirements, such as prestack migration. Furthermore, interpolation is also important for merging existing 3D datasets with different acquisition parameters so that the merged datasets have a uniform grid.

During the past decade or so, prestack 5-D interpolation became popular and was routinely used in processing. Many methods were developed to interpolate seismic traces in the 5-dimensional temporal-spatial domain. Minimum Weighted Norm Interpolation (MWNI) (Liu and Sacchi, 2004; Trad 2009), Projection Onto a Convex Set (POCS) (Abma and Kabir, 2006) and Anti-Leakage Fourier Transform (ALFT) (Xu et al., 2005; Xu et al., 2010) are Fourier transform based and work on the f-k^4 domain. Other methods, such as tensor completion (Trickett et al., 2013; Gao et al., 2015), are also used in the seismic processing world. For some practical considerations, the above methods work on a predefined grid either in the original data domain or in the transform domain, or both. Therefore, the locations of the input data and/or the true wavenumbers are snapped to the grid, which causes smearing of amplitude and phase information. To preserve amplitude and phase information, Arbitrarily Sampled Fourier Transform (ASFT) was developed (Guo et al., 2015; Zheng et al., 2015), which works in the continuous wavenumber domain and uses the true spatial coordinates.

Theory

The seismic data acquired from the field is considered to be in a 5-dimensional domain, one temporal dimension and four spatial dimensions. The 4D spatial domain can be (i) shot X-Y and receiver X-Y; (ii) inline, cross line, offset and azimuth; or (iii) inline, cross line, offset-X and offset-Y. The sampling interval in temporal dimension is uniform. However, the sampling intervals in the spatial domain are never uniform, due to the restrictions of the field conditions and financial constraint.

For Fourier transform based interpolation methods, each input seismic trace is first transformed from time to temporal frequency domain with Fast Fourier Transform (FFT). Then for each temporal frequency, the seismic data is transformed from 4D spatial domain to 4D wavenumber domain by FFT (MWNI, POCS), or Non-uniform Discrete Fourier Transform (NDFT) (ALFT). Interpolation is applied in the 4D wavenumber domain. After interpolation, the data is inverse transformed back to the 4D spatial domain and further to the time domain by 1D inverse FFT. All MWNI, POCS and ALFT use inverse FFT to transform data from the 4D wavenumber domain to the 4D spatial domain. Because the extent of the data in the 4D spatial domain varies, naturally, the wavenumber intervals are not uniform as well. Therefore, snapping the non-uniformly sampled wavenumber to a predefined grid might compromise the fidelity of the seismic data. To avoid any potential damage to the data, ASFT uses inverse NDFT to transform the data from the 4D wavenumber domain to the 4D spatial domain, which preserves the accuracy of Fourier components, i.e. the true positions of the wavenumbers and the strength of these wavenumbers.

Let *s(t, x)* be a collection of N seismic traces acquired from the field in a 5-dimensional domain, where *t* represents time and $\boldsymbol{x}$ is a 4-dimensional vector in space with N elements $\boldsymbol{x}_j$, *j*=1,…,N. Applying 1D FFT in time, the data is transformed to the temporal frequency domain by equation (1).

$$S(f, x) = FFT(s(t, x)) \tag{1}$$

Where f is the temporal frequency with a number of elements depending on the time window size.

To estimate the strength of each wavenumber, ASFT starts with a predefined grid, $\boldsymbol{k}$, with a total of M nodes, and calculates the Fourier coefficients of each $\boldsymbol{k}_m$. For each temporal frequency, f_i, 4-dimensional NDFT is applied to transform the data to the 4D wavenumber domain with equation (2).

$$W(f_i, k) = \sum_{j=1}^{N} g(x_j) S(f_i, x_j) e^{-2\pi i k \cdot x_j} \tag{2}$$

Where $g(\boldsymbol{x})$ is the weighting function related to the distribution of the traces in space. A proper weighting function will speed up the optimization process.

After estimating all possible wavenumbers, ASFT starts with one or a few of the strongest wavenumbers and optimizes their strength ($\boldsymbol{W}$) and position ($\boldsymbol{k}$), and then subtracts the optimized components from the original data. The next optimization is on the strongest Fourier components of the remaining data. This process repeats until the remaining data is below a user specified threshold.

The key factor differentiating ASFT from other methods is that ASFT not only optimizes the strength of the Fourier components, but also the positions of the wavenumbers so the quality of wave field reconstruction will not be compromised by the computation grid.

Once all Fourier components are solved for the optimized strengths ($\hat{\boldsymbol{w}}$) and positions ($\boldsymbol{k}_{op}$), inverse NDFT is applied to transform the optimized wave field back to the space domain, and inverse FFT is applied to transform data back to the time domain after summing up all temporal frequencies (equation 3). Note that $\boldsymbol{k}_{op}$ is not restricted to the predefined grid. It can be anywhere in the wavenumber domain.

$$\hat{s}(t, x_{reg}) = c\ FFT^{-1}(\sum_i\ (\sum_{op} \widehat{W}(f_i, k_{op}) e^{2\pi i x_{reg} \cdot k_{op}})) \quad (3)$$

Where $\hat{s}(t, x_{reg})$ is the reconstructed data at regularized locations in the time domain, $\widehat{W}(f_i, k_{op})$ is the optimized Fourier coefficients at the position $\boldsymbol{k}_{op}$ for the temporal frequency f_i, and c is a constant scalar.

Flexibility

One popular domain for interpolation is the azimuth-offset domain. It divides each offset into a number of azimuthal sectors, typically 6 or 8. A question often raised by clients is that when the azimuth-offset space is divided in such a way, each sector at near offset represents a much smaller area than a sector at far offset. In addition, the amplitude variation in azimuthal direction contains the anisotropic information and it varies more quickly at far offset. Therefore, it requires denser sampling at far offsets than near offsets.

Because ASFT solves Fourier components in the continuous wavenumber domain and use NDFT to transform the reconstructed data to the space domain, it has the flexibility to output reconstructed traces at any arbitrary locations. For example, more azimuthal sectors at far offset can be chosen to capture rapid changes of amplitude with little extra computation cost.

Examples

ASFT has been applied to a land seismic survey with an orthogonal geometry. The main purpose of interpolation is to minimize the footprints of the acquisition geometry and reduce the prestack migration artefacts. Figure 1 shows the comparison of time slices after PSTM without (left) and with (right) ASFT interpolation. ASFT interpolation reduced migration artefacts and provided a sharper and clearer image of the geological structures.

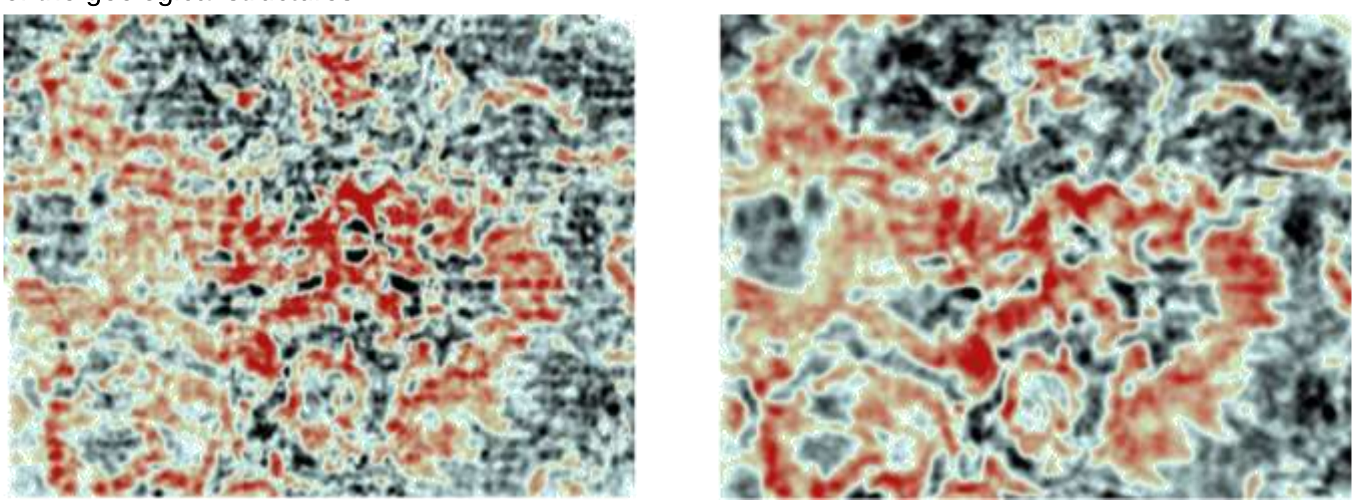

Figure 1. Comparison of time slices after PSTM without (left) and with (right) ASFT interpolation. ASFT reduced migration artefacts and provided sharper and clearer image of the geological structures.

As mentioned above, ASFT is able to output traces with variable number of azimuthal sectors at different offset in order to capture the rapid amplitude changes at far offsets. Figure 2 shows (a) the gather input to ASFT interpolation; (b) the gather after interpolation with 4 azimuthal sectors for all offsets; and (c) the gather after interpolation with 4 azimuthal sectors for near offsets (0 – 300 m), 8 azimuthal sectors for middle offsets (300 – 600 m) and 12 azimuthal sectors for far offset (600 – 900 m).

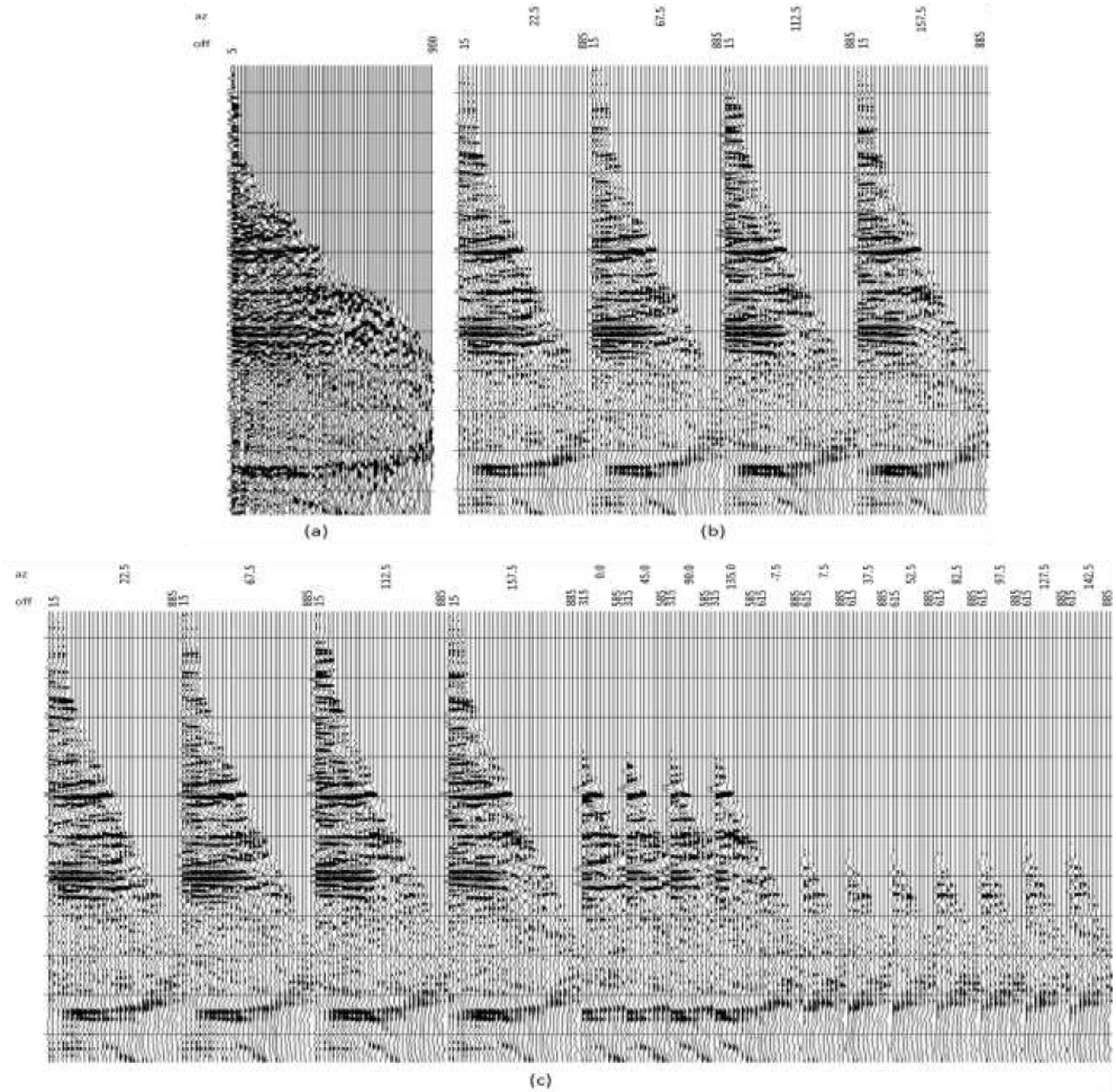

Figure 2. (a) input gather to ASFT; (b) ASFT output gather with 4 azimuthal sectors for all offsets; (c) ASFT output gathers with variable number of azimuthal sectors for different offset. In (c), the first 4 panels are the 4 azimuthal sectors for all offsets, the next 4 panels are the extra 4 azimuthal sectors for mid-offsets, and the last 8 panels are the extra 8 azimuthal sectors for far offsets.

Conclusions

Arbitrarily Sampled Fourier Transform (ASFT) optimizes both strengths and positions of the wavenumbers so it preserves the amplitude and phase information better. In addition, it can output to flexible locations, e.g. variable number of azimuthal sectors for different offset. Field data test shows ASFT minimized the acquisition footprint and produced sharper geological features on the PSTM results.

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